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DYNAMICAL MASS MATRICES FROM EFFECTIVE SUPERSTRING THEORIES[1]

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Abstract

We analyze the general structure of the fermion mass matrices in effective superstrings. They are generically given at low energy by non-trivial functions of the gauge singlet moduli fields. Interesting structures appear in particular if they are homogeneous functions of zero degree in the moduli. In this case we find Yukawa matrices very similar to the ones obtained by imposing a $U(1)$ family symmetry to reproduce the observed hierarchy of masses and mixing angles. The role of the $U(1)$ symmetry is played here by the modular symmetry. Explicit orbifold examples are given where realistic quark mass matrices can be obtained. Finally, a complete scenario is proposed which generates the observed hierarchies in a dynamical way.

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1 Introduction.

One of the outstanding problems of the Standard Model and its extensions is to understand the fermion masses and mixings. These are usually arbitrary parameters and the large mass hierarchy observed experimentally is still to be understood. Indeed, based on naive naturality arguments, the corresponding Yukawa couplings are expected to be all of order one . The explanation of the observed hierarchy certainly calls for new physics beyond the Standard Model.

The solutions to this problem proposed to this date fall essentially into two categories. One is a symmetry approach, first emphasized by Froggatt and Nielsen [1], which has been largely studied in the literature. It postulates a new abelian horizontal gauge symmetry spontaneously broken at a high energy scale M_X. The 3 families of quarks and leptons have different charges under the corresponding $U(1)_X$ group so that only a small number of the Standard Model Yukawa interactions be allowed by the symmetry $U(1)_X$. All the others appear through non-renormalisable couplings to a field whose vacuum expectation value $<\phi>$ breaks the horizontal symmetry. In the effective theory below the scale of breaking, this typically yields Yukawa couplings of the form

$$\lambda_{ij} = \left(\frac{<\phi>}{M_X} \right)^{n_{ij}}, \tag{1}$$

where n_{ij} depends on the $U(1)_X$ charges of the relevant fields. If $\varepsilon \equiv <\phi> / M_X$ is a small parameter, the hierarchy of masses and mixing angles is easily obtained by assigning different charges for different fermions.

A second approach, of dynamical origin, was recently proposed [2, 3, 4, 5, 6]. The main idea is to treat Yukawa couplings as dynamical variables to be fixed by the minimization of the vacuum energy density. In this case, one can show that a large hierarchy can be naturally obtained provided that the Yukawa couplings are subject to constraints. Such a constraint could be obtained by an ad hoc imposition of the absence of quadratic divergences in the vacuum energy [2, 5] or as an approximate infrared evolution of the renormalization group equations [4]. In ref. [6], a geometric origin for these constraints was proposed, related to the properties of the moduli space in effective superstring theories. In order to illustrate the idea, we give a simple example of a model containing two moduli fields T_1, T_2 and two fermions with moduli independent Yukawa couplings λ_1, λ_2. In this simple model the low energy couplings at the Planck scale M_P are simply computed to be

$$\hat{\lambda}_1 \sim \left(\frac{T_1 + T_1^+}{T_2 + T_2^+} \right)^{3/4} \lambda_1, \quad \hat{\lambda}_2 \sim \left(\frac{T_2 + T_2^+}{T_1 + T_1^+} \right)^{3/4} \lambda_2. \tag{2}$$

Thus $\hat{\lambda}_1$ and $\hat{\lambda}_2$ are homogeneous functions of zero degree in the moduli.

The moduli fields which describe the size and the shape of the six-dimensional compact manifold correspond to flat directions in the effective four-dimensional

supergravity theory. If these flat directions are exact, then the couplings in (2) can be regarded as dynamical variables to be determined by the low energy physics (much in the spirit of the no-scale idea [7] used in the dynamical determination of the gravitino mass [8]). It is easily seen however that the product

$$\hat{\lambda}_1 \hat{\lambda}_2 \sim \lambda_1 \lambda_2 \tag{3}$$

should be regarded as a constraint, because the moduli dependence has disappeared in the right hand side of (3). Minimization of the vacuum energy at a low energy scale with respect to the top and bottom Yukawa couplings subject to a constraint of the type (3) was studied in detail in [6]. It was shown there that, qualitatively, the ratio of the two couplings behaves as

$$\left(\frac{\lambda_b}{\lambda_t}\right)(\mu_0) \sim g^4(M_P)\frac{\mu}{M_{SUSY}}, \tag{4}$$

wher $\mu_0 \sim 1\ TeV$, $g(M_P)$ is the gauge coupling constant at the Planck scale, μ is the usual supersymmetric mass parameter of the MSSM and M_{SUSY} is the typical mass splitting between superpartners. For a large region of the parameter space of the MSSM, one can thus obtain $\lambda_t/\lambda_b \sim 40 - 50$ and easily fit the experimental masses with values of $\tan\beta$ of order 1.

The purpose of the present paper is twofold. First of all, we wish to show that in effective superstrings of the orbifold type [9], structures of the type (1) are naturally obtained. In this approach, the small parameter $\varepsilon =< \phi > /M_X$ of the $U(1)_X$ horizontal symmetry is given here by $\varepsilon = (T_1 + T_1^+)/(T_2 + T_2^+)$ in the case of two moduli. For n moduli, we obtain potentially $n-1$ small parameters and structures similar to the one given by a $[U(1)_X]^{n-1}$ horizontal symmetry.

The second goal is to show that one of the cases leading to these Froggatt-Nielsen structures[3] corresponds precisely to Yukawa couplings being homogeneous functions of the moduli. We can then apply the above-mentionned dynamical mechanism and determine by minimization the whole structure of the fermion matrices. The hierarchy translates into different vacuum expectation values of the moduli fields and different modular weights of the fermions with respect to these moduli.

Section 2 presents all the cases corresponding to Froggatt-Nielsen structures in orbifold-like effective models. In some instances, they appear when some – but not all – moduli fields are fixed to their self-dual values $T_i = 1$. An appealing situation is the case where the theory possesses a "diagonal" modular symmetry; then the Yukawa couplings are homogeneous functions of zero degree in the moduli and Froggatt-Nielsen structures appear even if all T_i are different from 1.

[3] From now on, we will call Froggatt-Nielsen structures mass matrices for which the order of magnitude of all entries are rational powers of a small common quantity ε.

Section 3 analyses, in analogy with the $U(1)_X$ approach, the relation between the mass matrices and the one-loop modular anomalies. It is shown that if there are no string threshold corrections in the gauge coupling constants the anomalies can be eliminated only by the Green-Schwarz mechanism [10] which uses the Kalb-Ramond antisymmetric tensor field present in superstring theories. In the case relevant for the dynamical approach, the modular anomalies can be cancelled by this mechanism only if there exists at least two moduli with modular anomalies cancelled by the Green-Schwarz mechanism. If the threshold corrections are present, they can account for a part of the modular anomalies and can provide a correct gauge coupling unification scenario, provided the modular weights of the Higgs fields satisfy a certain relation.

Section 4 deals with the dynamical determination of the mass matrices at low energy along the lines of ref. [6]. Two additional constraints on the modular weights are needed for the mechanism to be effective.

Section 5 is dedicated to a search of realistic orbifold models. It is found that in all the cases leading to Froggatt-Nielsen structures, no model can be constructed at Kac-Moody level one. Possible solutions exist at level two and three, even in the simplest case of only one small parameter. In the models with "diagonal" modular symmetry, we must appeal to more small parameters or, alternatively, go to higher Kac-Moody levels. Explicit examples with two small parameters at level three are given.

Some conclusions are presented at the end, together with open questions that remain to be investigated.

2 Low-energy mass matrices.

The low energy limit of the superstring models relevant for the phenomenology is the $N = 1$ supergravity described by the Kähler function K, the superpotential W and the gauge kinetic function f [11] . The generic fields present in the zero-mass string spectrum contain an universal dilaton-like field S, moduli fields generically denoted by T_α (which can contain the radii-type moduli T_α and the complex structure moduli U_β) and some matter chiral fields ϕ^i, containing the standard model particles. The Kähler potential and the superpotential read

$$\begin{aligned} K &= K_0 + \sum_i \prod_\alpha t_\alpha^{n_i^{(\alpha)}} |\phi^i|^2 + \cdots , \\ K_0 &= \hat{K}_0(T_\alpha, T_\alpha^+) - \ln(S + S^+), \\ W &= \frac{1}{3}\lambda_{ijk}\phi^i\phi^j\phi^k + \cdots , \end{aligned} \tag{5}$$

where the dots stand for higher-order terms in the fields ϕ^i. In (5), $t_\alpha = ReT_\alpha$ are the real parts of the moduli and $n_i^{(\alpha)}$ are called the modular weights of the

fields ϕ^i with respect to the modulus T_α . The λ_{ijk} are the Yukawa couplings which may depend nonperturbatively on S and T_α. We define the diagonal modular weight of the field ϕ^i as

$$n_i = \sum_\alpha n_i^{(\alpha)}. \tag{6}$$

An important role in the following discussion will be played by the target-space modular symmetries $SL(2,\mathbb{Z})$ associated with the moduli fields T_α , acting as

$$T_\alpha \to \frac{a_\alpha T_\alpha - ib_\alpha}{ic_\alpha T_\alpha + d_\alpha},\ a_\alpha d_\alpha - b_\alpha c_\alpha = 1,\ a_\alpha \cdots d_\alpha \in \mathbb{Z}\ . \tag{7}$$

In effective string theories of the orbifold type [9], the matter fields ϕ^i transform under (7) as

$$\phi^i \to (ic_\alpha T_\alpha + d_\alpha)^{n_i^{(\alpha)}} \phi^i \tag{8}$$

in order for the Kähler metric $K_i^j = \partial^2 K/\partial\phi^i\partial\bar{\phi}_j$ to be invariant.

This can be viewed as a particular type of Kähler transformations, which are symmetries of the supergravity theory. Denoting by z the set of the chiral fields, they read

$$\begin{aligned} K(z,z^+) &\rightarrow K(z,z^+) + F(z) + F^+(z^+), \\ W(z) &\rightarrow e^{-F(z)}\, W(z), \end{aligned} \tag{9}$$

where $F(z)$ is an analytic function.

A typical example is a model with n moduli fields and Kähler potential

$$K_0 = -\frac{3}{n}\sum_{\alpha=1}^{n} \ln(T_\alpha + T_\alpha^+) - \ln(S+S^+). \tag{10}$$

Under (7), it transforms as

$$K \to K + \frac{3}{n}\ln|ic_\alpha T_\alpha + d_\alpha|^2 \tag{11}$$

and the identification between eqs.(9) and (11) gives $F_\alpha = \frac{3}{n}\ln(ic_\alpha T_\alpha + d_\alpha)$. Associating a modular weight $n_{ijk}^{(\alpha)}$ with the trilinear couplings in eq.(5), the transformation (9) of W gives

$$n_i^{(\alpha)} + n_j^{(\alpha)} + n_k^{(\alpha)} + n_{ijk}^{(\alpha)} = -\frac{3}{n}. \tag{12}$$

Taking the sum of all such relations for the moduli fields, we find

$$n_i + n_j + n_k + n_{ijk} = -3. \tag{13}$$

The factor -3 in the right-hand side is related to the no-scale structure of the Kähler potential (10) and hence to the vanishing of the cosmological constant at tree level. More generally, $T^\alpha K_\alpha = -3 - V_0$, where V_0 is a nonzero contribution to the cosmological constant and the right-hand side is replaced by $-3 - V_0$. Eq.(13) is a weaker form of eqs.(12), expressing the invariance of the theory under the diagonal modular transformations with respect to all moduli :

$$\phi^i \to \prod_\alpha (ic_\alpha T_\alpha + d_\alpha)^{n_i^{(\alpha)}} \phi^i. \tag{14}$$

The difference between the individual modular transformations and the less restrictive diagonal one will be essential in the following.

The low energy spontaneously broken theory contains the canonically normalized field $\hat{\phi}^i$ defined by $\phi^i = (K^{-1/2})^i_j \ \hat{\phi}^j$ and the Yukawas $\hat{\lambda}_{ijk}$ which give the physical masses. The matching condition at the Planck scale M_p relating the low energy and the original Yukawa couplings is

$$\hat{\lambda}_{ijk} = e^{\frac{K_0}{2}} (K^{-1/2})^{i'}_i (K^{-1/2})^{j'}_j (K^{-1/2})^{k'}_k \lambda_{i'j'k'} \ . \tag{15}$$

From eq.(15) we see that the $\hat{\lambda}_{ijk}$ are functions of the moduli through the Kähler potential K and eventually the $\lambda_{i'j'k'}$.

Our goal is to analyze the general structure of the mass matrices for the quarks and leptons as a function of the moduli fields. They are described by the superpotential $\hat{W}$ of the Minimal Supersymmetric Standard Model (MSSM) which we take to be the minimal model obtained in the low-energy limit of the superstring models, plus eventually some extra matter singlet under the Standard Model gauge group. $\hat{W}$ contains the Yukawa interactions

$$\hat{W} \supset \hat{\lambda}^U_{ij} \hat{Q}^i \hat{U}^c_j \hat{H}_2 + \hat{\lambda}^D_{ij} \hat{Q}^i \hat{D}^c_j \hat{H}_1 + \hat{\lambda}^L_{ij} \hat{L}^i \hat{E}^c_j \hat{H}_1, \tag{16}$$

where H_1 and H_2 are the two Higgs doublets of MSSM , Q^i, L^i are the $SU(2)$ quark and lepton doublets and U_j, D_j , E_j are the right-handed $SU(2)$ singlets.

Consider the case of two moduli T_1 and T_2. Using eqs. (5), (6), (15) and (16) , a U-quark coupling reads

$$\hat{\lambda}^U_{ij} = e^{\frac{K_0}{2}} \ t_2^{-\frac{n_{Q_i} + n_{U_j} + n_{H_2}}{2}} \left(\frac{t_1}{t_2}\right)^{-\frac{n^{(1)}_{Q_i} + n^{(1)}_{U_j} + n^{(1)}_{H_2}}{2}} \lambda^U_{ij} \tag{17}$$

or equivalently,

$$\hat{\lambda}^U_{ij} = e^{\frac{K_0}{2}} \ t_1^{-\frac{n_{Q_i} + n_{U_j} + n_{H_2}}{2}} \left(\frac{t_2}{t_1}\right)^{-\frac{n^{(2)}_{Q_i} + n^{(2)}_{U_j} + n^{(2)}_{H_2}}{2}} \lambda^U_{ij} \ . \tag{18}$$

Suppose that one of the two moduli-dependent factors in (17) (or equivalently (18)) happens to be family blind. Then the structure obtained for the Yukawa

matrix turns out to be very similar to the one that would be derived from an horizontal $U(1)$ symmetry of the Froggatt-Nielsen type [1]. Modular weights play the role of the $U(1)$ charges. Such a situation may arise in the following three cases of interest:

i) $t_1 = t_2 = t \neq 1$.

Then

$$\hat{\lambda}^U_{ij} = t^{-\frac{n_{Q_i}+n_{U_j}+n_{H_2}+3}{2}} \lambda^U_{ij} \ , \tag{19}$$

where n_{Q_i}, etc are the diagonal modular weights defined in eq. (6). For $t << 1$ this could produce hierarchical Yukawa couplings. This case is disfavoured in the case where the relation (13) holds with $n_{ijk} = 0$.

ii) $t_2 = 1, \frac{t_1}{t_2} = \varepsilon << 1$ or vice versa $t_1 \leftrightarrow t_2$. Then using eq.(17) we get

$$\hat{\lambda}^U_{ij} \sim \varepsilon^{-\frac{n^{(1)}_{Q_i}+n^{(1)}_{U_j}+n^{(1)}_{H_2}}{2}} \lambda^U_{ij}, \tag{20}$$

where we dropped the universal $e^{\frac{K_0}{2}}$ factor, irrelevant here. Hierarchical structures are obtained if the dynamics imposes $\varepsilon = \frac{t_1}{t_2}$ small (typically of the order of the Cabibbo angle to some power). Remark that the relevant modular weights correspond to the modulus whose ground state falls away from the self-dual points, $t_i \neq 1$ (for an example of such a situation, see Ref.[12]).

iii) one has the condition

$$n_{Q_i} + n_{U_j} + n_{H_2} = \text{ independent of } \ i \text{ and } j. \tag{21}$$

and $\frac{t_1}{t_2} = \varepsilon << 1$ or vice-versa. This obviously implies that $n_{Q_i} = n_Q$ and $n_{U_i} = n_U$ for any $i = 1, 2, 3$.

For example, in the case when the diagonal modular symmetry holds, the constant (21) is equal to -3 and the Yukawa couplings can be written as

$$\hat{\lambda}^U_{ij} = \frac{1}{S+\bar{S}} \left(\frac{t_1}{t_2}\right)^{-\frac{n^{(1)}_{Q_i}+n^{(1)}_{U_j}+n^{(1)}_{H_2}+\frac{3}{2}}{2}} \lambda^U_{ij} \ = \ \frac{1}{S+\bar{S}} \left(\frac{t_2}{t_1}\right)^{-\frac{n^{(2)}_{Q_i}+n^{(2)}_{U_j}+n^{(2)}_{H_2}+\frac{3}{2}}{2}} \lambda^U_{ij} \tag{22}$$

or

$$\hat{\lambda}^U_{ij} = \frac{1}{S+\bar{S}} \left(\frac{t_1}{t_2}\right)^{-\frac{n^{(12)}_{Q_i}+n^{(12)}_{U_j}+n^{(12)}_{H_2}}{4}} \lambda^U_{ij} \tag{23}$$

where $n^{(12)}_{Q_i} = n^{(1)}_{Q_i} - n^{(2)}_{Q_i}$, etc. The last form (23) is particularly useful in that it relates the Froggatt-Nielsen-like structures with the asymmetry between the modular weights corresponding to the two moduli fields. [4]

[4] Such formulas show that the magnitude of the modular weights must appear in a hierarchy

One way to get the condition (21) is to search for models where the $\hat{\lambda}_{ijk}$ are homogeneous functions of the moduli T_α, i.e. $\sum_\alpha T_\alpha \partial \hat{\lambda}_{ijk}/\partial T_\alpha = 0$. In this case, using the relation $\sum_\alpha t_\alpha \partial K_i^j/\partial t_\alpha = n_i K_i^j$ and the matching condition (15), we arrive at an equation for the original couplings λ_{ijk}

$$(\frac{1}{2}T_\alpha K^\alpha - \frac{n_i+n_j+n_k}{2} + T_\alpha \frac{\partial}{\partial T_\alpha})\lambda_{ijk} = 0. \tag{25}$$

If $T_\alpha K^\alpha = -3$ and the λ_{ijk} are pure numbers we recover eq.(13). In such a case ($n_{ijk}=0$), the relation (25) can be derived from assuming the diagonal modular symmetry discussed above in (14). This approach was used in [6] in a dynamical approach to the fermion mass problem proposed in [2], [3] and studied in [4] and [5]. We will return to it in section 4.

The experimental data on the fermion and the mixing angles can be summarized as follows. Defining $\lambda = sin\theta_c \sim 0.22$ where θ_c is the Cabibbo angle, the mass ratios and the Kobayashi-Maskawa matrix elements at a high scale $M_X \sim M_P$ have the values

$$\begin{gathered}\tfrac{m_u}{m_t} \sim \lambda^7 \text{ to } \lambda^8 \ , \ \tfrac{m_c}{m_t} \sim \lambda^4 \ , \ \tfrac{m_d}{m_b} \sim \lambda^4 \ , \ \tfrac{m_s}{m_b} \sim \lambda^2 \ , \\ \tfrac{m_e}{m_\tau} \sim \lambda^4 \ , \ \tfrac{m_\mu}{m_\tau} \sim \lambda^2 \ , \ |V_{us}| \sim \lambda \ , \ |V_{cb}| \sim \lambda^2 \ , \ |V_{ub}| \sim \lambda^3 \text{ to } \lambda^4. \end{gathered} \tag{26}$$

Taking as a small parameter $\varepsilon = \lambda^2 \sim \frac{1}{20}$, these values are perfectly acommodated by the following modular weight assignement

$$\begin{aligned} n_{Q_3}^{(1)} - n_{Q_1}^{(1)} = 3 \ , \ n_{Q_3}^{(1)} - n_{Q_2}^{(1)} = 2 \\ n_s^{(1)} - n_b^{(1)} = 0 \ , \ n_b^{(1)} - n_d^{(1)} = 1 \\ n_t^{(1)} - n_c^{(1)} = 2 \ , \ n_t^{(1)} - n_u^{(1)} = 4 \end{aligned} \tag{27}$$

corresponding to the mass matrices

$$\hat{\lambda}_U = \lambda^{-x} \begin{pmatrix} \lambda^7 & \lambda^5 & \lambda^3 \\ \lambda^6 & \lambda^4 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix} \ , \ \ \hat{\lambda}_D = \lambda^y \begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda & 1 & 1 \end{pmatrix} . \tag{28}$$

which is opposite for the two moduli. In fact, the construction of models with two moduli along these lines leads to a difficulty compared to the case i). This is because in order to get the required hierarchy, for example

$$\hat{\lambda}_{ij}^U / \hat{\lambda}_{33}^U \ \sim \ \varepsilon^{-\frac{n_{Q_i}^{(1)} - n_{Q_3}^{(1)} + n_{U_j}^{(1)} - n_{U_3}^{(1)}}{2}} \ \lambda_{ij}^U / \lambda_{33}^U, \tag{24}$$

we need $|n_{Q_i}^{(1)} - n_{Q_3}^{(1)} + n_{U_j}^{(1)} - n_{U_3}^{(1)}| >> 1$. But the condition (21) implies $n_{Q_i}^{(1)} - n_{Q_3}^{(1)} + n_{U_j}^{(1)} - n_{U_3}^{(1)} = -\left(n_{Q_i}^{(2)} - n_{Q_3}^{(2)} + n_{U_j}^{(2)} - n_{U_3}^{(2)}\right)$ so a large asymmetry of the modular weights for the moduli T_1 should be compensated by a large asymmetry, of opposite sign, coming from the modular weights of the moduli T_2. This is difficult to satisfy and asks generally for more that two moduli or (and) higher Kac-Moody levels, as we will see later in explicit constructions. But the construction above is easily generalized to the case of 3 or more moduli.

In (28) $x \geq 0$ (the top coupling should be at least of order one at a high scale) and $y \geq 0$ (the bottom coupling should correspondingly be smaller or equal to one). Remark that the negative power of λ in $\hat{\lambda}_U$ is impossible to obtain in a horizontal symmetry approach because of the analyticity of the superpotential. In the moduli case it comes naturally and it plays an important role in a dynamical approach to the fermion masses, as will be shown in section 4.

A detailed analysis in the case of an horizontal $U(1)$ symmetry shows that other assignments which fit approximately the values (26) are possible [13] , but the one considered here is the best suited for our purposes. This is because the others ask for larger modular weight differences, which are difficult to get in realistic orbifold models. This topic will be discussed in greater detail in section 5. The expressions (28) will be the starting point in the construction of abelian orbifold models with realistic mass matrices.

An interesting aspect of the case iii) discussed above should be stressed which concerns the sfermion masses $(M_0^2)_{i\bar{j}}$. Non-diagonal sfermion mass matrices give potentially dangerous contributions to flavor changing neutral current processes like $b \to s\gamma$ or $\mu \to eJ\gamma$ [14]. The general expression in supergravity is

$$(M_0^2)_{i\bar{j}} = (M_{1/2}M_{1/2}^+)_{i\bar{j}} + (G_{i\bar{j}} - G_\alpha R^\alpha_{\bar{j}i\beta}G^\beta)m_{3/2}^2, \tag{29}$$

where $G = K + ln|W|^2$ and $G_{i\bar{j}} = \frac{\partial^2 G}{\partial\phi^i \partial\phi^+_{\bar{j}}}$ is the metric on the Kähler space. The indices α, β correspond to moduli fields which contribute to supersymmetry breaking $< G_\alpha > \neq 0$ and we assume $< G_\alpha G^\alpha >= 3$. (This corresponds to the moduli limit in the language of refs [15], [16] where the universal dilaton does not contribute to supersymmetry breaking. The general case is irrelevant for the present analysis). $R^\alpha_{\bar{j}i\beta}$ is the Riemann tensor of the Kähler space, $(M_{1/2})_{ik}$ the fermion mass matrix and $m_{3/2}$ the gravitino mass. Using the intermediate formula

$$G_\alpha R^\alpha_{\bar{j}i\beta}G^\beta = t^\alpha t^\beta \frac{\partial^2 G_{i\bar{j}}}{\partial t^\alpha \partial t^\beta} - \frac{\partial G_{i\bar{k}}}{\partial t^\alpha}(G^{-1})^{\bar{k}l}\frac{\partial G_{l\bar{j}}}{\partial t^\beta} = -n_i G_{i\bar{j}} \tag{30}$$

and doing a trivial rescaling in order to define the low-energy parameters, we find

$$\begin{aligned}
(M_0^2)^{U,D}_{LL} &= m_{3/2}^2(1 + n_Q)\mathbb{1} \ + (M_{1/2}M_{1/2}^+)^{U,D}, \\
(M_0^2)^U_{RR} &= m_{3/2}^2(1 + n_U)\mathbb{1} \ + (M_{1/2}M_{1/2}^+)^U, \\
(M_0^2)^D_{RR} &= m_{3/2}^2(1 + n_D)\mathbb{1} \ + (M_{1/2}M_{1/2}^+)^D,
\end{aligned} \tag{31}$$

where $(M_0^2)^{U,D}_{LL}$ is the left-left squark squared mass respectively for the U and the D quark, etc. In (31) n_Q, n_U and n_D are the diagonal modular weights which in the case iii) discussed above are the same for the three generations. The important aspect of (31) is that the squark soft-breaking mass matrix is

proportional to the identity. Going to the basis where the quark mass matrices are diagonal, we see that the soft squark masses are still proportional to the unit matrix. Consequently, there are no flavor changing neutral currents induced at the supergravity level. Even if we know by now [15] that SUGRA - induced flavor changing neutral currents are not as severe as thought several years ago, it is still worth emphasizing the virtue of the case iii), corresponding to considering Yukawa as being homogeneous functions of the moduli.

3 Modular anomalies and moduli mass textures.

In the context of horizontal abelian symmetries used to explain fermion mass hierarchies, an interesting connection has been established [17, 18] between anomalies associated with such symmetries and mass hierarchies as given in (26).

Gauge anomalies in usual field theories must be absent in order to define a consistent quantum theory. In effective supergravity theories the gravitational anomalies must cancel too in order to preserve the reparametrization invariance. This requirement imposes non-trivial constraints on the particle spectrum in chiral theories. Applied to the ten-dimensional superstrings, this led to the famous Green-Schwarz anomaly cancellation mechanism involving the Kalb-Ramond antisymmetric tensor field [10]. This mechanism has a counterpart in 4 dimensions which allows to fix the value of $\sin^2\theta_W$ at the string scale without advocating a grand unified symmetry [19]. It was shown in [18] and generalized in [13, 20] that, using this mechanism, it is possible to infer from the observed hierarchies (26) in the mass matrices the standard value of $3/8$ for $\sin^2\theta_W$.

Effective string models also have another type of anomalies, named σ-model anomalies [21]. They appear, as in the gauge case, in triangle diagrams with two gauge bosons and one modulus and have two different origins: one is the nontrivial metric of the matter fields; the other non-invariance can be analyzed as a violation of the Kähler invariance of the SUGRA theory i.e. under transformations of the type (7). It is known that modular transformations are symmetries to all orders in the string perturbation theory. Moreover, the string massive spectrum is separately invariant, as can be checked by interchanging the Kaluza-Klein and the winding states. Thus it is within the zero mass spectrum of the string, which defines the effective SUGRA theory, that the corresponding triangle anomalies must cancel at the field theory level. We will show in this section that the cancellation of these anomalies plays a role very similar to the one of mixed gauge anomalies in the abelian horizontal $U(1)_X$ symmetry approach.

Consider the non-linear σ-model corresponding to the moduli fields, generically denoted by T_β, which refers as above both to the Kähler (radii-type) and complex structure moduli. The gauge group is $G = \prod_a G_a$ and there are matter fields in different representations R_a of G_a. The anomalous triangle diagrams give a non-local contribution to the one-loop effective lagrangian [22] which reads

$$\begin{aligned}\mathcal{L}_{nl} &= \frac{1}{8}\frac{1}{16\pi^2}\sum_a \int d^4\theta (W^\alpha W_\alpha)_a \frac{\mathcal{D}^2}{\Box} \ \Big(C(G_a) K(T_\beta, T_\beta^+) \\ &\quad + \sum_{R_a} T(R_a) \left[2 \ln \det K_{i\bar{j}}^{R_a}(T_\beta, T_\beta^+) - K(T_\beta, T_\beta^+) \right] \Big) + h.c. \qquad (32)\end{aligned}$$

In eq.(32) where superfield notations are used, W^α is the Yang-Mills field strength superfield ; $K_{i\bar{j}}^{R_a}$ is the Kähler metric for the matter fields in the representation R_a of group G_a.

The cases of interest to be analyzed in this paper are orbifold compactifications. Consider the diagonal Kähler moduli for which $\hat{K}_0(T_\beta, T_\beta^+) = -\ln(T_\beta + T_\beta^+)$ (and possibly the complex structure moduli). Then the above expressions reduce to

$$\mathcal{L}_{nl} = \frac{1}{8}\frac{1}{16\pi^2}\sum_a \int d^4\theta (W^\alpha W_\alpha)_a \frac{\mathcal{D}^2}{\Box} \sum_\beta b_a^{\prime(\beta)} \ln(T_\beta + T_\beta^+) + h.c. \qquad (33)$$

where the anomaly coefficients $b_a^{\prime(\beta)}$ are given by the expressions

$$b_a^{\prime(\beta)} = -C(G_a) + \sum_{R_a} T(R_a)(1 + 2n_{R_a}^{(\beta)}). \qquad (34)$$

One finds that the change of $\mathcal{L}_{nl}$ under the modular transformations (7) is given by the local expression

$$\delta\mathcal{L}_{nl} = \frac{1}{2}\frac{1}{16\pi^2}\sum_a \int d^2\theta (W^\alpha W_\alpha)_a \sum_\beta b_a^{\prime(\beta)} \ln(ic_\beta T_\beta + d_\beta) + h.c. \qquad (35)$$

There are two ways of compensating this anomaly. The first, which is particularly interesting in our case is reminiscent of the Green-Schwarz mechanism. It uses the form of the tree level gauge kinetic term

$$\mathcal{L}_{tree} = \sum_a \int d^2\theta \frac{1}{4} k_a S (W^\alpha W_\alpha)_a + h.c. \qquad (36)$$

and requires the non-invariance of the dilaton field under the modular transformations

$$S \to S - \frac{1}{8\pi^2}\sum_\beta \delta_{GS}^{(\beta)} \ln(ic_\beta T_\beta + d_\beta). \qquad (37)$$

The factor $\delta_{GS}^{(\beta)}$ is the gauge group independent Green-Schwarz coefficient and induces a mixing between the dilaton S and the moduli fields T_β. This mechanism can completely cancel the anomalies only if the anomaly coefficients $b_a'^{(\beta)}$ satisfy the equalities

$$\delta_{GS}^{(\beta)} = \frac{b_a'^{(\beta)}}{k_a} = \frac{b_b'^{(\beta)}}{k_b} = \cdots \tag{38}$$

for all the group factors of the gauge group $G = \prod_a G_a$.

A second mechanism for the cancellation of the term (35) uses the one-loop threshold corrections to the gauge coupling constants, which can be different for different gauge group factors. They appear for the moduli fields associated with complex planes left unrotated by some twist vectors and are due to contributions from the massive Kaluza-Klein and winding states. If the modular symmetry group is $(SL(2,\mathbb{Z}))^3$, the one-loop running gauge coupling constants at a scale μ reads

$$\frac{1}{g_a^2(\mu)} = \frac{k_a}{g_s^2} + \frac{b_a}{16\pi^2} ln \frac{M_s^2}{\mu^2} - \frac{1}{16\pi^2} \sum_{\alpha=1}^{3} (b_a'^{(\alpha)} - k_a \delta_{GS}^{(\alpha)}) \ln \left[(T_\alpha + T_\alpha^+) |\eta(T_\alpha)|^4 \right] \tag{39}$$

In (39) g_s is the string coupling constant, M_s is the string scale and b_a are the RG β-function coefficients ($a = 1, 2, 3$) for $U(1)_Y$, $SU(2)_L$ and $SU(3)$ respectively. The Dedekind function is defined by $\eta(T) = exp(-\pi T/12) \prod_{n=1}^{\infty} [1 - exp(2\pi n T)]$, which transforms under (7) as $\eta(T_\alpha) \to \eta(T_\alpha)(ic_\alpha T_\alpha + d_\alpha)^{\frac{1}{2}}$. The unification scale M_U is determined by the condition

$$k_1 g_1^2(M_U) = k_2 g_2^2(M_U) = k_3 g_3^2(M_U), \tag{40}$$

and computed to be

$$M_U = M_s \prod_{\alpha=1}^{3} \left[(T_\alpha + T_\alpha^+) |\eta(T_\alpha)|^4 \right]^{\frac{b_b'^{(\alpha)} k_a - b_a'^{(\alpha)} k_b}{2(b_a k_b - b_b k_a)}} . \tag{41}$$

with $a \neq b \in \{1, 2, 3\}$. In the simplest approximation of neglecting all the threshold corrections, a one-loop RG analysis for g_3 and $sin^2\theta_W$ gives a good agreement with the experimental data if $M_U \simeq M_s/50$ [23].

Consider now a minimal orbifold model with the particle content of the MSSM (respectively Q_i, U_i, D_i, L_i, i being a family index, and the two Higgs supermultiplets H_1 and H_2), plus possibly extra Standard Model singlet fields. The mixed Kähler $SU(3) \times SU(2) \times U(1)_Y$ triangle anomalies are described by the coefficients [24]

$$b_1'^{(\beta)} = 11 + \sum_{i=1}^{3} (\frac{1}{3} n_{Q_i}^{(\beta)} + \frac{8}{3} n_{U_i}^{(\beta)} + \frac{2}{3} n_{D_i}^{(\beta)} + n_{L_i}^{(\beta)} + 2 n_{E_i}^{(\beta)}) + n_{H_1}^{(\beta)} + n_{H_2}^{(\beta)},$$

$$
\begin{aligned}
b_2'^{(\beta)} &= 5+\sum_{i=1}^{3}(3n_{Q_i}^{(\beta)}+n_{L_i}^{(\beta)})+n_{H_1}^{(\beta)}+n_{H_2}^{(\beta)}, \\
b_3'^{(\beta)} &= 3+\sum_{i=1}^{3}(2n_{Q_i}^{(\beta)}+n_{U_i}^{(\beta)}+n_{D_i}^{(\beta)}).
\end{aligned} \tag{42}
$$

Apart from the modular weight independent piece, these coefficients are identical to the ones encountered for the mixed $U(1)_X - G_a$ gauge group anomalies in the abelian horizontal $U(1)_X$ gauge symmetry approach. Again, the role of the $U(1)_X$ charges is played here by the modular weights of the different fields. Consequently we will closely follow the analysis performed in [17] , [18] and [13].

We first place ourselves in the case ii) of the preceding section, where only one modular weight is relevant for the mass matrices. In what follows, the modular weights and the anomaly coefficients refer to only one of the moduli. As noted in Ref. [18] in the case of an horizontal abelian symmetry, an interesting consequence of eqs.(20) and (42) is

$$
\frac{Det\hat{\lambda}_L}{Det\hat{\lambda}_D} \sim \varepsilon^{2+\frac{n_{H_1}+n_{H_2}}{2}-\frac{1}{4}(b_1'+b_2'-\frac{8}{3}b_3')} \ . \tag{43}
$$

Also, as in the analysis of [13, 20], we have

$$
(\mathrm{Det}\hat{\lambda}_U)(\mathrm{Det}\hat{\lambda}_L)^3(\mathrm{Det}\hat{\lambda}_D)^{-2} \sim \varepsilon^{-\frac{3}{4}(b_1'+b_2'-2b_3'-10)} \ . \tag{44}
$$

This equation has the advantage not to contain the unknown variable $n_{H_1}+n_{H_2}$, which allows to draw general conclusions. It is clear from (44) that for realistic mass values and phenomenologically acceptable ratio λ_b/λ_t we cannot put the anomalies to zero $b_1'=b_2'=b_3'=0$.

In the case where there are no threshold corrections associated with the modulus giving the mass structures, the only solution is the use of the Green-Schwarz mechanism [17, 18]

$$
b_2'=b_3'=\frac{3}{5}b_1'=b' \tag{45}
$$

corresponding to Kac-Moody level ratios

$$
k_1:k_2:k_3=\frac{5}{3}:1:1. \tag{46}
$$

As is well known, this leads to the successful prediction of the Weinberg angle at a high scale M_U, $sin^2\theta_W=\frac{3}{8}$ [19]. Using the experimental input (26) which can be expressed as $Det\hat{\lambda}_L \sim Det\hat{\lambda}_D \sim \lambda^{6+3y}$, $Det\hat{\lambda}_U \sim \lambda^{12-3x}$, and taking $\epsilon \sim \lambda^2$, eqs.(43) , (44) and (45) fix the variables

$$
\begin{aligned}
n_{H_1}+n_{H_2} &= -4, \\
b' &= -3+3(x-y),
\end{aligned} \tag{47}
$$

where x and y are defined in (28) .

If there are threshold corrections in the modulus field giving the Froggatt-Nielsen structures, then eq. (43) can be rewritten in the form (assuming the Kac-Moody levels (46))

$$\frac{\mathrm{Det}\hat{\lambda}_L}{\mathrm{Det}\hat{\lambda}_D} \sim \varepsilon^{2+\frac{n_{H_1}+n_{H_2}}{2}} (M_U/M_s)^{\frac{1}{2}(b_1+b_2-\frac{8}{3}b_3)\ ln\varepsilon\left[ln(T_\alpha+T_\alpha^+)|\eta(T_\alpha)|^4\right]^{-1}} \ . \tag{48}$$

The ratio of the two logarithms in the right-hand side is easy to compute for $t_\alpha \sim \varepsilon$. In this case, $(T+T^+)|\eta(T)|^4 \sim \frac{2}{\varepsilon}e^{\frac{-\pi}{3\varepsilon}}$. Consequently the ratio of the two logarithms is approximately $-3\varepsilon ln\varepsilon/\pi \sim 1/4$. Because of modular invariance a similar value is obtained for $t_\alpha \sim \varepsilon^{-1}$. For the phenomenologically relevant case $M_U \sim M_s/50$, the successful relation $\mathrm{Det}\hat{\lambda}_L = \mathrm{Det}\hat{\lambda}_D$ asks for

$$n_{H_1} + n_{H_2} \simeq -10 \ . \tag{49}$$

This is an interesting possibility which will be further investigated in section 5. Using the possible modular weights of the Higgs fields at Kac-Moody levels 2 and 3 from the tables 1 and 2 and the known renormalization group coefficients b_a, we can find the allowed values of the unification scale M_U. As a result, as we will discuss in section 5, it is found that the relation (49) can be satisfied. Thus, the threshold corrections contribute in order to give a good unification scheme as well as realistic mass textures.

Eqs. (49) and (47) must be taken into account in the construction of explicit models. Generically it is clear, by a simple inspection of eqs.(42) that we need many fields in the twisted sectors of the orbifolds in order to satisfy eqs. (49) and (47). Twisted fields are in any case necessary in order to get the assignement (27) leading to the desired hierarchical structure.

We now turn to the case (iii) of the preceding section. Starting from the relation (23), we obtain, using the same technique as before:

$$\begin{aligned}
(\mathrm{Det}\hat{\lambda}_U)(\mathrm{Det}\hat{\lambda}_L)^3(\mathrm{Det}\hat{\lambda}_D)^{-2} &\sim \varepsilon^{-\frac{3}{8}(b_1^{\prime(12)}+b_2^{\prime(12)}-2b_3^{\prime(12)})}, \\
\frac{Det\hat{\lambda}_L}{Det\hat{\lambda}_D} &\sim \varepsilon^{-\frac{1}{8}[b_1^{\prime(12)}+b_2^{\prime(12)}-\frac{8}{3}b_3^{\prime(12)}-2(n_{H_1}^{(12)}+n_{H_2}^{(12)})]} \ .
\end{aligned} \tag{50}$$

The first of eqs. (50) is very useful to discuss anomaly cancellation conditions. Taking as an example $\varepsilon \sim \lambda^m$, it requires

$$b_1^{\prime(1)} + b_2^{\prime(1)} - 2b_3^{\prime(1)} = b_1^{\prime(2)} + b_2^{\prime(2)} - 2b_3^{\prime(2)} - \frac{48}{m}. \tag{51}$$

As shown in [18] for the case of an horizontal symmetry, the second of eqs. (50) has the following interesting solution, which automatically gives the value 3/8 for $\sin^2\theta_W$ at unification:

$$\begin{aligned}
n_{H_1}^{(1)} + n_{H_2}^{(1)} &= n_{H_1}^{(2)} + n_{H_2}^{(2)}, \\
b_1^{\prime(1)} + b_2^{\prime(1)} - \frac{8}{3}b_3^{\prime(1)} &= b_1^{\prime(2)} + b_2^{\prime(2)} - \frac{8}{3}b_3^{\prime(2)}.
\end{aligned} \tag{52}$$

Moreover, using the conditions (13) in the case $n_{ijk}=0$ and the expressions (42), we obtain

$$\begin{aligned} b'_1 + b'_2 - 2b'_3 &= 8, \\ b'_1 + b'_2 - \frac{8}{3}b'_3 &= 2(8 + n_{H_1} + n_{H_2}), \end{aligned} \tag{53}$$

where $b'_1 = b'^{(1)}_1 + b'^{(2)}_1$, etc. Eqs. (51) and (53) clearly express the fact that the theory has one-loop modular anomalies.

An analysis of all the possibilities for the anomalies related to the two moduli leads to the conclusion that, without threshold corrections, the mixed case with zero anomalies for one modulus and Green-Schwarz mechanism for the other modulus is physically uninteresting (it requires $\varepsilon \sim \lambda^{\pm 6}$). In the case of anomalies cancelled by the Green-Schwarz mechanism for both moduli, we obtain $n^{(1)}_{H_1} + n^{(1)}_{H_2} = n^{(2)}_{H_1} + n^{(2)}_{H_2} = -4$ and $b'^{(i)} = 6(1 \pm 6/m)$ for $i = 2, 1$. The only other allowed case is when threshold corrections are present for both moduli. In this case, we obtain $n_{H_1} + n_{H_2} + 8 = \frac{60\lambda^m}{\pi} \ln(M_S^2/M_U^2)$. A realistic value for M_U requires $m \leq 2$ and is obtained for example for $m = 2$, $n_{H_1} + n_{H_2} = -14$. Let us note that $\sin^2\theta_W$ can still be found equal to 3/8 at unification scale, irrespective of the choices made in order to obtain the desired value for M_U.

4 Dynamical determination of couplings.

The duality symmetries imply the existence of flat directions in the corresponding moduli fields. If they are respected to all orders in the supergravity interactions, then the only way to lift them is by breaking supersymmetry. Given the scale expected for this breaking, one may expect the low energy sector to play an important role in the determination of the moduli ground state. Under these conditions, the low energy minimization with respect to the moduli fields is presumably equivalent to the minimization with respect to the Yukawa couplings, through their non-trivial dependence on the moduli. This was the attitude taken in Refs. [4, 5, 6] to dynamically determine the top/bottom Yukawa couplings. A very important point in this program is the existence of constraints between Yukawas , of a type which is typical of the approach based on moduli dynamics. It was shown in Ref. [6] that this can be enforced if the Yukawa couplings are homogeneous functions of the moduli. In what follows, we will therefore place ourselves in the case iii) of section 2 and analyze how the two approaches can be merged, leading to a dynamical determination of the fermion mass hierarchies and mixing angles.

We start by reviewing the results of Ref. [6]. To compute the vacuum energy at the low-energy scale $\mu_0 \sim M_{susy}$ we proceed in the usual way. Using boundary

values compatible with the constraints at the Planck scale M_P (identified here with the unification scale), we evolve the running parameters down to the scale μ_0 using the RG equations and adopt the effective potential approach [25]. The one-loop effective potential has two pieces

$$V_1(\mu_0) = V_0(\mu_0) + \Delta V_1(\mu_0) \ , \tag{54}$$

where $V_0(\mu_0)$ is the renormalization group improved tree-level potential and $\Delta V_1(\mu_0)$ summarizes the quantum corrections given by the formula

$$\Delta V_1(\mu_0) = (1/64\pi^2) \ Str\mathcal{M}^4 \ (\ln \frac{\mathcal{M}^2}{\mu_0^2} - \frac{3}{2}) \ . \tag{55}$$

In (55) $\mathcal{M}$ is the field-dependent mass matrix, $Str\mathcal{M}^n = \sum_J (-1)^{2J} (2J + 1) \ TrM_J^n$ is the ponderated trace of the mass matrix for particles of spin J and all the parameters are computed at the scale μ_0. The vacuum state is determined by the equation $\partial V_1 / \partial \phi_i = 0$, where ϕ_i denotes collectively all the fields of the theory. The vacuum energy is simply the value of the effective potential computed at the minimum.[5]

As expected there is no Yukawa coupling dependence at the tree level. At the one-loop level it appears through

$$\frac{1}{3} \ Str\mathcal{M}^4 = A_U Tr\lambda_U^2 + A_D Tr(\lambda_D^2 + \frac{1}{3}\lambda_L^2) + 8\mu Tr(\lambda_U \mathcal{A_U} + \lambda_D \mathcal{A_D} + \frac{1}{3}\lambda_L \mathcal{A_L}) v_1 v_2 \ , \tag{56}$$

where v_1 and v_2 are the vacuum expectation values of the two Higgs doublets. In (56) $\mathcal{A_U}$, $\mathcal{A_D}$ and $\mathcal{A_L}$ are trilinear soft breaking terms and the trace is in the family space. A_U and A_D are given by the expressions

$$\begin{aligned} A_U &= 2 \ [2\mu^2 / tg^2\beta + 4M^2 - M_Z^2 + (g_1^2 + g_2^2) v_1^2] \ v_2^2 \ , \\ A_D &= 2 \ [2\mu^2 \ tg^2\beta + 4M^2 - M_Z^2 + (g_1^2 + g_2^2) v_2^2] \ v_1^2 \ , \end{aligned} \tag{57}$$

where g_1, g_2 are the $U(1)$, $SU(2)$ gauge couplings, M is a universal squark soft mass and $M_Z^2 = \frac{1}{2}(g_1^2 + g_2^2)(v_1^2 + v_2^2)$ is the Z mass. In order to show that $A_U, A_D > 0$, one may use the phenomenological inequality

$$(Str\mathcal{M}^2)_{\text{quarks + squarks}} = 4M^2 > M_Z^2 \ . \tag{58}$$

The vacuum energy (54) has roughly the Nambu form [2] with an additional linear term which does not change the shape of the vacuum energy as a function of the Yukawas, but which plays an essential role in the minimization process.

The positivity of A_U, A_D is a consequence of supersymmetry in the sense that it is due to the Yukawa dependent bosonic contributions in (56). In the

[5] In a first approximation, if the moduli masses are larger than the average superpartner mass $\tilde{m}$, the factor $\ln \mathcal{M}^2/\mu_0^2$ in (55) can be replaced by $\ln \tilde{m}^2/\mu_0^2 < 0$. [5, 6]

non-supersymmetric Standard Model the sign is negative and the present considerations do not apply. Using eq.(54) and eq.(55), we obtain the vacuum energy as a function of the matrices λ_U and λ_D, which is a paraboloid unbounded from below. If the minimization is freely performed, then they are driven to the maximally allowed values and no hierarchy is generated.

Consider now the mass matrices (28) with $\lambda \sim (t_1/t_2)^{\frac{1}{2}}$ a dynamical parameter to be determined by the minimization. We discussed in Ref.[6] two types of constraints: (a) the proportionality constraint where one of the couplings is proportional to another (to some positive power) $\lambda_1 = \text{cst} \cdot \lambda_2^n,\ n > 0$, (b) a *multiplicative constraint* where the product of two couplings (or positive powers of them) is fixed to be a moduli independent constant: $\lambda_1 \lambda_2^n = \text{cst},\ n > 0$. Only the second constraint leads to dynamical hierarchy of couplings. Fortunately for $x, y > 0$ in (28) we get the second type of constraints, for example $(\hat\lambda_U^{33})^y (\hat\lambda_D^{33})^x = cst$. In this case if $\hat\lambda_U^{33}$ for example is big, the constraint (valid at M_P) forces $\hat\lambda_D^{33}$ to be small and we naturally obtain small numbers.

For the case of two moduli, the conditions to have $x > 0$, $y > 0$ read

$$n_{Q_3} + n_{U_3} + n_{H_2} > -3/2\ , n_{Q_3} + n_{D_3} + n_{H_1} < -3/2 \tag{59}$$

and they should be fulfilled in order to obtain multiplicative-type constraints. An interesting case (treated in detail in [6], where we keeped only λ_U^{33} and $\hat\lambda_D^{33}$ in the computations) is $x = y$. The relevant constraints are then symmetric in the up and down quarks.

The low energy effective potential is to be minimized with respect to λ. For this the RG equations are used in order to translate the structures (28) from M_P to μ. The analysis is essentially the same as in [6], the whole structure of the mass matrices does not change qualitatively the results. There are essentially two conditions for the top quark to be the heaviest fermion. The first is (for $g_1 = 0$)

$$tg^2\beta > \frac{2M^2 + m_1^2}{2M^2 + m_2^2}\ . \tag{60}$$

where m_1, m_2 are the supersymmetric mass terms for the two Higgs. The second is a rather involved lower bound for the dilaton vacuum expectation value, so that the underlying string theory must be in a perturbative regime. We therefore need a minimal critical value for $tg\beta$ of order one, which depends on the soft masses, in order to have a heavy top quark. Under these two assumptions, there is no need of fine tuning to obtain a value of λ of order 0.2 which allows to understand the hierarchy between the top quark and the other fermions.

5 Search for realistic orbifold examples.

We saw in Section 2 that a wide spread of modular weights is needed in order to reproduce the mass matrix hierarchies. In the case of orbifold compactifications, modular weights are computable numbers for each given model. We consider here the case of abelian symmetric orbifolds and evaluate which class of models is selected by the requirement of the preceding section. We will closely follow the approach of Ref. [24].

The Kähler potential of the matter fields has a simple dependence on the three generic moduli T_α and on three model-dependent fields U_m. The corresponding modular weights are denoted by $n_i^{(\alpha)}$ and $l_i^{(\alpha)}$.

The spectrum fields consists of two sectors :

• The untwisted sector, corresponding to the string boundary conditions $(i = 1 \cdots 6)$

$$X^i(\sigma = 2\pi, t) = X^i(\sigma = 0, t) + V^i, \tag{61}$$

where V^i are shifts in the six-dimensional lattice obtained by the action of the space group.

• The twisted sector, corresponding to the string boundary conditions

$$X^i(\sigma = 2\pi, t) = \theta X^i(\sigma = 0, t) + V^i, \tag{62}$$

where θ is a twist which is an automorphism of the six-dimensional lattice (some discrete rotation) $\theta^N = 1$, where N is called the order of the twist.

In order to have $N = 1$ supersymmetry, θ must belong to $SU(3)$. The twist θ can be generally written as

$$\theta = \exp 2\pi i[v_1 J_{12} + v_2 J_{34} + v_3 J_{56}], \tag{63}$$

where $Nv_i \in \mathbb{Z}$ and the J_{mn} are the $SO(6)$ Cartan generators. $N = 1$ supersymmetry implies $\pm v_1 + \pm v_2 + \pm v_3 = 0$ for some choice of signs.

A generic twisted oscillator state has the form

$$\prod_{\beta,\gamma=1}^{3} \prod_{m_\beta, n_\gamma} (\alpha^\beta_{m_\beta+\theta_\beta})^{p^\beta_m} (\widetilde{\alpha}^\gamma_{n_\gamma-\theta_\gamma})^{q^\gamma_n} |vacuum>, \tag{64}$$

where α^β and $\widetilde{\alpha}^\gamma$ are the creation operators corresponding respectively to the analytic and antianalytic oscillators in the compactified directions. The indices

m_β and n_γ are the orders of the corresponding oscillators. Defining the total number of oscillators as $p^\beta = \sum_m p^\beta_m$ and $q^\gamma = \sum_n q^\gamma_n$, the modular weights of the oscillator states are, for the twisted sector,

$$n_i^{(\alpha)} = -(1-\theta^\alpha + p_i^\alpha - q_i^\alpha) \ , l_i^{(\alpha)} = -(1-\theta^\alpha + q_i^\alpha - p_i^\alpha) \text{ if } \theta^\alpha \neq 0 \ , \qquad (65)$$

$$n_i^{(\alpha)} = l_i^{(\alpha)} = 0 \ \text{ if } \theta^\alpha = 0 \ .$$

The states in the untwisted sector are characterized by

$$n_\beta^{(\alpha)} = -\delta^\alpha_\beta \ , l_\beta^{(\alpha)} = -\delta^\alpha_\beta \ . \qquad (66)$$

The relevant quantity to evaluate the mass matrices is $n_i^{(\alpha)} - n_j^{(\alpha)}$. The states i and j can be either in the untwisted or in the twisted sector. We are searching for states such that $n_i^{(\alpha)} - n_j^{(\alpha)}$ is maximum in order to obtain the asymmetric relations eqs.(27). If, for example, both states i and j are in the sector described by the same twist vector, we obtain by using eq. (65)

$$n_i^{(\alpha)} - n_j^{(\alpha)} = p_j^{(\alpha)} + q_i^{(\alpha)} - \big(p_i^{(\alpha)} + q_j^{(\alpha)}\big). \qquad (67)$$

Since the low energy particles are to be found in the massless spectrum of the string, we are a priori interested only in the modular weights of the massless particles. Some constraints can be obtained from the mass formula for the left-moving twisted states

$$\frac{1}{8} M_L^2 = N_{osc} - h_{KM} + E_0 - 1 \qquad (68)$$

where N_{osc} is the fractional oscillator number. E_0 is the zero-point energy of the twist θ given by the formula

$$E_0 = \sum_{\alpha=1}^{3} \frac{1}{2} |v^\alpha| (1 - |v^\alpha|), \qquad (69)$$

where v^α is defined in eq.(63). The constant h_{KM} is the contribution to the conformal dimension of the matter fields from the left-moving $E_8 \times E_8$ gauge part. If the gauge group is $G = \prod_a G_a$ and the massless particles are in the representations R_a of G_a, then

$$h_{KM} = \sum_a \frac{C(R_a)}{C(G_a) + k_a} \ , \qquad (70)$$

where k_a is the Kac-Moody level of the gauge factor G_a. In the following, we consider a model containing the spectrum of the MSSM and possibily extra

matter or (and) gauge interactions. Therefore h_{KM} computed within the MSSM gives a lower bound to the real value. The minimum number of the oscillator states is given by

$$p_{\max} \leq N(1 - E_0 - h_{KM}) \ , \tag{71}$$

$$q_{\max} \leq \frac{1}{1-\theta^j} \ (1 - E_0 - h_{KM}) \ , \tag{72}$$

where N is the order of the twist $\overrightarrow{\theta}$.

Armed with these results we can readily check that the relations (27) are impossible to satisfy at the Kac-Moody levels $k_2 = k_3 = \frac{3}{5}k_1 = 1$.

At level two, a complete scan of the abelian orbifolds gives the results displayed in Table 1.

The most difficult relation to satisfy, which is therefore our main concern, is $n_t^{(\alpha)} - n_u^{(\alpha)} = 4$, for some complex plane α. In table 1, only two models approximatively satisfy it, the $\mathbb{Z}_{12}$ and $\mathbb{Z}'_{12}$ orbifolds with $(|v_1|, |v_2|, |v_3|) = (\frac{1}{3}, \frac{1}{12}, \frac{5}{12})$ and $(\frac{1}{2}, \frac{1}{12}, \frac{5}{12})$ respectively. The right-handed up quark should be in the first or the fifth twisted sector $\theta(\theta^{12} = 1)$ and $(p, q) = (3, 0)$ with respect to the second or the third complex plane. The top right-handed quark is in the untwisted sector with $n_u^{(2)} = 0$. In this example $t_1 = t_3 = 1$, $\varepsilon = t_2/t_3 \sim (0.22)^2$. The second and the third complex planes are completely rotated by the twist vectors so the modular anomalies related to it must be cancelled completely by the Green-Schwarz mechanism.

More possibilities are allowed at Kac-Moody level three, as seen in Table 2. The orbifolds which can accommodate the hierarchy are

- $\mathbb{Z}_8$ and $\mathbb{Z}'_8$ of twists $(\frac{1}{2}, \frac{1}{8}, \frac{3}{8})$ and $(\frac{1}{4}, \frac{1}{8}, \frac{3}{8})$, respectively. In both cases the right-handed up quark should be in the first or the third twisted sector and $(p, q) = (3, 0)$ with respect to the second or the third complex plane. The right-handed top quark is in the untwisted sector with $n_t^{(2)} = 0$. The anomalies with respect to the second and the third planes are completely cancelled by the Green-Schwarz mechanism.

- $\mathbb{Z}_{12}$ and $\mathbb{Z}'_{12}$ of twists $(\frac{1}{3}, \frac{1}{12}, \frac{5}{12})$ and $(\frac{1}{2}, \frac{1}{12}, \frac{5}{12})$. The right-handed up quark must be in the first or the fifth twisted sector, but more assignments for (p, q) can be given. The right-handed top quark can be in the untwisted or in the twisted sector. Another possibility for the up-quark is the second twisted sector of $\mathbb{Z}'_{12}$ of twist $(0, \frac{1}{6}, \frac{5}{6})$. The anomalies with respect to the second and the third planes are completely cancelled by the Green-Schwarz mechanism.

- $\mathbb{Z}_2 \times \mathbb{Z}_6$, $\mathbb{Z}_3 \times \mathbb{Z}_6$ and $\mathbb{Z}_6 \times \mathbb{Z}_6$ of twists $(0, \frac{1}{6}, \frac{5}{6})$. The example is similar to the $\mathbb{Z}_8$, $\mathbb{Z}'_8$ cases, with the exception of the anomalies. Here all of the three

complex planes are left unrotated by a particular twist. Consequently there are threshold corrections in the gauge coupling constants which can partially cancel the modular anomalies. Moreover, because of the fact that $p_{max}^{H_1,H_2} = 3$, the relation (49) can be satisfied. so these models have the possibility of accomodating a phenomenologically correct unification scale M_U.

We also display in Table 3 an example for the case (iii) of Section 2, which uses two small parameters $t_2/t_1 \sim \lambda,\ t_1/t_3 \sim \lambda$. Only the oscillators for the right-handed up-quarks are displayed, the others being easy to obtain.

As a general rule, the higher the Kac-Moody level, the simpler it is to get mass hierarchies due to a wider spread of the allowed modular weights. Such models have recently received attention in an attempt of constructing grand unified string theories [26, 27].

In general, the hierarchy appears as follows. For a modulus corresponding to a small parameter, the second family fermions should have more string oscillators compared to the third family and the first family more than the second one (the opposite being true for a modulus corresponding to a large parameter). The hierarchy thus translates into a decreasing number of allowed oscillators when going from the light to the heavy families.

6 Concluding remarks.

In this paper we analyzed the structure of the fermion mass matrices in the effective superstring theories. It is found that, in some cases of phenomenological interest, they are similar to the structures obtained by imposing abelian horizontal symmetries. The analog of the abelian charges are the modular weights of the matter fields; the small expansion parameters are provided by the vev's of some moduli fields away from their self-dual values. Hierarchical structures for the mass matrices are obtained by assigning different modular weights for the three families of quarks and leptons with respect to some moduli fields. A particular case of interest is when the Yukawas are homogeneous functions of the moduli, which can be viewed as a consequence of a 'diagonal' modular symmetry of the theory, in the case where the original string couplings are pure numbers. An interesting consequence is that the squark and slepton mass matrices are proportional to the identity matrix. Consequently they give no contributions to the FCNC processes like $b \to s\gamma$ or $\mu \to e\gamma$.

We stressed an intriguing connection between the mass matrices and the modular anomalies, similar to the one between mass matrices and mixed gauge anomalies in the horizontal symmetry approach recently discussed in the literature. A phenomenologically relevant mass spectrum requires one-loop modular anomalies, which can be cancelled in two ways. The first one is the Green-Schwarz mechanism of superstrings. In this context, if the Yukawa couplings are homogeneous functions of moduli and if the sum of the modular weights

of the two Higgs doublets of the MSSM is symmetric in the moduli, then a correct mass pattern asks for a Green-Schwarz mechanism with $k_1 = \frac{5}{3}$ and the Weinberg angle is predicted to be $\sin^2\theta_W = \frac{3}{8}$. The second way uses the moduli dependent threshold corrections to the gauge coupling constants. In this case we obtain a relation between the fermion masses, modular weights and the unification scale M_U. Our analysis shows that we can acommodate a low value $M_U \sim M_s/50$ provided the Higgs modular weights satisfy a constraint which is allowed at Kac-Moody level two or three in abelian orbifolds. Hence we have the possibility of a succesful unification scheme.

We have also investigated a dynamical mechanism for understanding the fermion masses as a low-energy minimization process, previously restricted to the top and bottom couplings. We show that the mechanism is easily generalized to account for the whole structure of the mass matrices, provided two inequalities on the modular weights hold.

We have given orbifold examples where the hierarchies of the type that we propose are allowed. There are no examples at Kac-Moody level one due to the limited range of the allowed modular weights, but we give examples at level two and three.

There are, of course, many open questions and problems which deserve further investigations. First of all the vev's of the moduli fields should be fixed by the dynamics, which usually prefers the self-dual points. In the dynamical approach, it would be also interesting to view the determination of the Yukawa couplings directly from the point of view of the moduli fields: in particular why the corresponding flat directions remain unlifted down to low energies.

Finally it would be interesting to construct explicit orbifold models with hierarchical mass matrices along these lines and to investigate their phenomenological virtues.

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Table 1. Maximum number of allowed oscillators in abelian orbifolds for $(3/5)k_1 = k_2 = k_3 = 2$.

E_0	$\lvert v_1\rvert, \lvert v_2\rvert, \lvert v_3\rvert$	$h^Q_{KM} = \frac{37}{80}$	$h^U_{KM} = \frac{2}{5}$	$h^D_{KM} = \frac{3}{10}$	$h^{H_1,H_2}_{KM} = \frac{21}{80}$
0	$(0,0,0)$	$(p,q) = (0,0)$	$(0,0)$	$(0,0)$	$(0,0)$
$\frac{1}{3}$	$(\frac{1}{3}, \frac{1}{3}, \frac{2}{3})$	$(0,0)$	$(0,0)$	$(1,0)$	$(1,0)$
$\frac{5}{16}$	$(\frac{1}{2}, \frac{1}{4}, \frac{1}{4})$	$(0,0)$	$(1,0)$	$(1,0)$	$(1,0)$
$\frac{1}{4}$	$(\frac{1}{3}, \frac{1}{6}, \frac{1}{6})$	$(1,0)$	$(2,1)$	$(2,1)$	$(2,1)$
$\frac{11}{36}$	$(\frac{1}{2}, \frac{1}{3}, \frac{1}{6})$	$(1,0)$	$(1,0)$	$(2,0)$	$(2,0)$
$\frac{14}{49}$	$(\frac{3}{7}, \frac{2}{7}, \frac{1}{7})$	$(1,0)$	$(2,0)$	$(2,0)$	$(3,1)$
$\frac{19}{64}$	$(\frac{1}{2}, \frac{1}{8}, \frac{3}{8})$	$(1,0)$	$(2,0)$	$(3,0)$	$(3,0)$
$\frac{17}{64}$	$(\frac{1}{4}, \frac{1}{8}, \frac{3}{8})$	$(2,0)$	$(2,0)$	$(3,1)$	$(3,1)$
$\frac{13}{48}$	$(\frac{1}{3}, \frac{1}{12}, \frac{5}{12})$	$(3,0)$	$(3,0)$	$(5,1)$	$(5,1)$
$\frac{41}{144}$	$(\frac{1}{2}, \frac{1}{12}, \frac{5}{12})$	$(3,0)$	$(3,0)$	$(5,0)$	$(5,0)$
$\frac{1}{4}$	$(0, \frac{1}{2}, \frac{1}{2})$	$(0,0)$	$(0,0)$	$(0,0)$	$(0,0)$
$\frac{2}{9}$	$(0, \frac{1}{3}, \frac{1}{3})$	$(0,0)$	$(1,1)$	$(1,1)$	$(1,1)$
$\frac{3}{16}$	$(0, \frac{1}{4}, \frac{1}{4})$	$(1,1)$	$(1,1)$	$(2,2)$	$(2,2)$
$\frac{5}{36}$	$(0, \frac{1}{6}, \frac{1}{6})$	$(2,2)$	$(2,2)$	$(3,3)$	$(3,3)$

Table 2. Maximum number of allowed oscillators in abelian orbifolds for $(3/5)k_1 = k_2 = k_3 = 3$.

E_0	$\|v_1\|, \|v_2\|, \|v_3\|$	$h^Q_{KM} = \frac{17}{45}$	$h^U_{KM} = \frac{14}{45}$	$h^D_{KM} = \frac{11}{45}$	$h^{H_1,H_2}_{KM} = \frac{1}{5}$
0	$(0,0,0)$	$(p,q) = (0,0)$	$(0,0)$	$(0,0)$	$(0,0)$
$\frac{1}{3}$	$(\frac{1}{3},\frac{1}{3},\frac{2}{3})$	$(0,0)$	$(1,0)$	$(1,0)$	$(1,0)$
$\frac{5}{16}$	$(\frac{1}{2},\frac{1}{4},\frac{1}{4})$	$(1,0)$	$(1,0)$	$(1,0)$	$(1,0)$
$\frac{1}{4}$	$(\frac{1}{3},\frac{1}{6},\frac{1}{6})$	$(2,1)$	$(2,1)$	$(3,1)$	$(3,1)$
$\frac{11}{36}$	$(\frac{1}{2},\frac{1}{3},\frac{1}{6})$	$(1,0)$	$(2,0)$	$(2,0)$	$(2,0)$
$\frac{14}{49}$	$(\frac{3}{7},\frac{2}{7},\frac{1}{7})$	$(2,0)$	$(2,0)$	$(3,1)$	$(3,1)$
$\frac{19}{64}$	$(\frac{1}{2},\frac{1}{8},\frac{3}{8})$	$(2,0)$	$(3,0)$	$(3,0)$	$(4,1)$
$\frac{17}{64}$	$(\frac{1}{4},\frac{1}{8},\frac{3}{8})$	$(2,0)$	$(3,1)$	$(3,1)$	$(4,1)$
$\frac{13}{48}$	$(\frac{1}{3},\frac{1}{12},\frac{5}{12})$	$(4,0)$	$(5,1)$	$(5,1)$	$(6,1)$
$\frac{41}{144}$	$(\frac{1}{2},\frac{1}{12},\frac{5}{12})$	$(4,0)$	$(5,0)$	$(5,0)$	$(6,1)$
$\frac{1}{4}$	$(0,\frac{1}{2},\frac{1}{2})$	$(0,0)$	$(0,0)$	$(1,1)$	$(1,1)$
$\frac{2}{9}$	$(0,\frac{1}{3},\frac{}{3})$	$(1,1)$	$(1,1)$	$(1,1)$	$(1,1)$
$\frac{3}{16}$	$(0,\frac{1}{4},\frac{1}{4})$	$(1,1)$	$(2,2)$	$(2,2)$	$(2,2)$
$\frac{5}{36}$	$(0,\frac{1}{6},\frac{1}{6})$	$(2,2)$	$(3,3)$	$(3,3)$	$(3,3)$

Table 3. $\mathbb{Z}_{12}$ orbifold example for case (iii) with three moduli and two small parameters, at Kac-Moody level three.

quark	twisted sector	$(p^{(1)}, q^{(1)})$	$(p^{(2)}, q^{(2)})$	$(p^{(3)}, q^{(3)})$
u	θ	$(0, 0)$	$(4, 0)$	$(0, 0)$
c	θ^2	$(1, 0)$	$(2, 0)$	$(1, 0)$
t	θ^5	$(0, 0)$	$(0, 0)$	$(4, 0)$

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